

Formulate the dual for each of the following optimization problems.

**Problem 1**

$$\begin{aligned} & \text{maximize } 4x_1 + x_2 \\ & \text{subject to } x_1 + 2x_2 \leq 5 \\ & \quad -2x_1 + x_2 \geq 1 \\ & \quad 3x_1 - x_2 = 2 \\ & \quad x_1, x_2 \geq 0. \end{aligned}$$

**Problem 2**

$$\begin{aligned} & \text{minimize } 3x_1 + 2x_2 + x_3 \\ & \text{subject to } x_1 + 4x_2 - x_3 \geq 2 \\ & \quad 2x_1 - x_2 + 3x_3 \geq 3 \\ & \quad x_1 \geq 0, \quad x_2 \text{ free}, \quad x_3 \geq 0. \end{aligned}$$

**Problem 3**

$$\begin{aligned} & \text{maximize } 2x_1 + 5x_2 \\ & \text{subject to } 3x_1 + x_2 \leq 8 \\ & \quad x_1 + 4x_2 \leq 9 \\ & \quad 0 \leq x_1 \leq 2, \quad 0 \leq x_2 \leq 3. \end{aligned}$$

**Problem 4**

$$\begin{aligned} & \text{maximize } -x_1 + 3x_2 \\ & \text{subject to } -x_1 + x_2 \leq 2 \\ & \quad 2x_1 + x_2 = 1 \\ & \quad x_1 \leq 0, \quad x_2 \geq 0. \end{aligned}$$

**Problem 5**

Let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 4 \\ 5 \end{bmatrix}, \quad c = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}.$$

Consider

$$\begin{aligned} & \text{maximize } c^\top x \\ & \text{subject to } Ax \leq b, \\ & \quad x \geq 0. \end{aligned}$$

**Problem 6**

$$\begin{aligned} & \text{minimize } 5x_1 - x_2 + 2x_3 \\ & \text{subject to } -x_1 + 2x_2 + x_3 \geq 3 \\ & \quad 4x_1 + x_2 - 2x_3 \geq 1 \\ & \quad x_1 \geq 0, \quad x_2 \leq 0, \quad x_3 \text{ free.} \end{aligned}$$

**Problem 7**

$$\begin{aligned} & \text{minimize } 0 \\ & \text{subject to } 2x_1 - x_2 \geq 1, \\ & \quad -x_1 + 3x_2 \geq 2, \\ & \quad x_1, x_2 \geq 0. \end{aligned}$$

**Problem 8**

$$\begin{aligned} & \text{maximize } x_1 - 2x_2 + x_3 \\ & \text{subject to } x_1 + x_2 + x_3 = 4, \\ & \quad 2x_1 - x_2 + 3x_3 = 1, \\ & \quad x_1 \geq 0, \quad x_2 \text{ free}, \quad x_3 \leq 0. \end{aligned}$$