CME 307: Optimization	CME 307 / MS&E 311 / OIT 676

Quiz 8: Practice questions

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Exercise 0.1. Let $f(x) = \|x\|_2$, the Euclidean norm (not squared). Using the fact that convex functions are differentiable almost everywhere and that subgradients arise as limits of gradients along differentiable points, compute $\partial f(0)$. Give a short justification based on directional derivatives or gradient limits.

Exercise 0.2. For a symmetric matrix $X \in \mathbb{R}^{n \times n}$, the function $f(X) = \lambda_{\max}(X)$ has subgradients

$$\partial f(X) = \operatorname{conv}\{vv^\top : |v| = 1,\ Xv = \lambda_{\max}(X)v\}.$$

Compute $\partial f(X)$ explicitly for

$$X = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Exercise 0.3. Let

$$f(x) = \max\{x_1 + 2x_2,\ 3x_1 - x_2,\ -x_1 + 4x_2\}.$$

At $x = (1, 1)$, determine which affine terms are active and compute the full subdifferential $\partial f(x)$.

Exercise 0.4. For

$$f(x) = \delta_{[0,\infty)}(x) = \begin{cases} 0 & x \geq 0 \\ \infty & x < 0 \end{cases},$$

the Fenchel conjugate is $f^*(y) = \sup_{x \geq 0}(xy)$. Compute $f^*(y)$ and then use the Fenchel relation

$$g \in \partial f(x) \iff f(x) + f^*(g) = xg$$

to determine $\partial f(0)$.

Exercise 0.5. Minimize $f(x) = |x| + 5$ starting from $x_0 = 3$. Perform one step of the subgradient method with step size $\alpha = 0.4$. Because the subgradient at 0 is a set, list all possible next iterates.

Exercise 0.6. Let $C = \{x \in \mathbb{R}^2 : 0 \leq x \leq 1\}$, where the inequality holds elementwise. The proximal operator of its indicator is the projection $\operatorname{prox}_{1_C}(w) = \operatorname{proj}_C(w)$. Compute this projection for $w = (3, 4)$.

Exercise 0.7. Let

$$f(x) = \tfrac{1}{2}(x - 5)^2 + 1_{[0,\infty)}(x).$$

Compute $\operatorname{prox}_{tf}(v)$ for $v = 1$ and $t = 1$. (You may treat the indicator by restricting the minimization to $x \geq 0$.)

Exercise 0.8. Consider $f(x) = |x - 2|$

$$T(x) = \operatorname{prox}_f(x).$$

Find all fixed points of T. Interpret them in terms of the optimality condition $0 \in \partial f$.